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Callan–Symanzik equations and low-energy theorems with trace anomalies

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Abstract

Based on some new and concise forms of the Callan–Symanzik equations, the low-energy theorems involving trace anomalies à la Novikov–Shifman–Vainshtein–Zakharov, first advanced and proved in Novikov *et al* (1980 *Nucl. Phys.* B **165** 67, 1981 *Nucl. Phys.* B **191** 301), are proved as immediate consequences. The proof is valid in any consistent effective field theories and these low-energy theorems are hence generalized. Some brief discussions about related topics are given.

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Introduction

It is well known that the powerful low-energy theorems à la Novikov–Shifman–Vainshtein–Zakharov (NSVZ) [1, 2] have played very important roles in the early studies of nonperturbative QCD and physical properties of hadrons[1]. These theorems are crucial also for many recent investigations concerning the nonperturbative components of QCD (see, e.g., [3–5] and the references therein). However, although these low-energy theorems involve the scale or trace anomalies [6], the connection of these theorems with the Callan–Symanzik equations (CSE), which are known to be comprehensive or complete accounts for the trace anomalies and renormalization issues, is not obvious in the original proofs given in [1, 2]. In fact, as already noted in [2], in the elegant proof first given in [1] the renormalization issue was 'simply' ignored. In the relatively more sophisticated treatment of the renormalization issue in appendix B of [2], the proof of the low-energy theorems was given in terms of an appropriate nonlocal correlation function using a special regularization and yet its connection with CSE was not established.

[1] See, e.g., the references listed in [2].

In this paper, I wish to propose a simple and general proof of the low-energy theorems given in [1, 2, 7], directly based on the CSE in a form that is more illuminating in comprehending the full account of trace anomalies arising from renormalization. Specifically, I wish to demonstrate the utility of the new versions of the Callan–Symanzik equations grounded upon the concept of effective field theories [8] and the *intrinsic* connections between CSE and these low-energy theorems.

In [8], a simple derivation of the renormalization group equations (RGE) and CSE (merely based on the standard concept of effective field theories) was proposed, with a new interpretation of running in terms of the decoupling effects of the underlying structures. Such an approach or parametrization could provide us with a simple and hence transparent comprehension of all possible ways that regularization/renormalization can affect the field theoretical computations. Also given there were various forms of the RGE and CSE, suitable for different purposes. As will be clear shortly, the low-energy theorems under consideration just come from one version of CSE. In fact, due to the general validity of CSE, these low-energy theorems should be valid in any consistent field theories besides QCD. Therefore, these low-energy theorems involving trace anomalies are generalized in this sense.

Renormalization of EFT and underlying structures

To obtain the appropriate forms of CSE, we briefly recall some reasonings and deductions of [8]. According to the standard viewpoint, all the known quantum field theories are only effective frameworks for dominant modes within a certain range of scales, with the structures or modes at far separated scales (i.e., underlying the effective ones) being 'ignored'. The price paid for this 'ignorance' is the unphysical ultraviolet (UV) and/or infrared (IR) singularities. Of course, given a complete theoretical description where the underlying structures are properly formulated, no such pathology should appear. In this sense, we could view all the necessary regularization and renormalization/factorization procedures as some sort of substitutes or 'representations' of the underlying theory's description of the effective field theory (EFT) sectors.

Since the complete theory is unavailable, our simplest speculation of the underlying theory (UT)'s description of the effective modes might be the addendum of the underlying parameters or constants ($[\sigma]$) to that of EFT ($[g]$) in the *finite or renormalized* Green functions $\Gamma^{\cdots}([p],[g;\sigma])$, or, equivalently, in the *finite or renormalized* generating functional or path integral

$$\begin{aligned} Z([J]_{\mathrm{EFT}};[g;\sigma]) &\equiv \int D\mu_{\mathrm{UT}}\exp(\mathrm{i}\mathcal{S}_{\mathrm{UT}}([g;\sigma];[J]_{\mathrm{EFT}})) \\ &= \int D\mu_{\mathrm{EFT}}\exp(\mathrm{i}\mathcal{S}_{\mathrm{EFT}}([g];[J]_{\mathrm{EFT}})+\mathrm{i}\Delta\mathcal{S}([g;\sigma];[J]_{\mathrm{EFT}})). \end{aligned} \tag{1}$$

Obviously, the extra action $\Delta\mathcal{S}([g;\sigma];[J])]$ contains all the necessary details that makes the description well defined, in contrast to the simplified EFT framework. This very extra action, or the very underlying structures, is responsible for all the possible anomalies in the EFT terminology. In the following, we are mainly concerned with scale or trace anomaly.

Since $\sum_g d_g g\partial_g$ (from now on, $d_{\{\ldots\}}$ denotes the canonical scale dimension of the corresponding parameter or constant) induces the insertion of the canonical trace of the energy–momentum tensor of an EFT, $-\mathrm{i}\int \mathrm{d}^D x\Theta(x)$, then it is straightforward to see that the 'canonical' piece $\sum_\sigma d_\sigma\sigma\partial_\sigma$ in the underlying theory is the only source of the trace anomalies to EFT, when it is expanded in terms of the effective field operators, or, when the underlying scales are taken to be infinite (large or small, the EFT limit). Thus it is convenient to introduce the canonical trace for the underlying theory in the following way,

$$\tilde{\Theta} \equiv \Theta + \Delta\Theta \Leftrightarrow \mathrm{i}\left\{\sum_g d_g g \partial_g + \sum_\sigma d_\sigma \sigma \partial_\sigma\right\}, \tag{2}$$

with $\Delta\Theta \Leftrightarrow \mathrm{i}\sum_\sigma d_\sigma \sigma \partial_\sigma$ denoting the 'canonical' underlying component, which shall appear as trace anomalies in terms of EFT parameters. Conventionally, these anomalies are attributed to the consequences of renormalization procedures. Here, we could view the latter as an effective 'representation' (or substitute) of the true underlying structures' contributions. We stress again that $\tilde{\Theta}$ is canonical in terms of the complete framework of the underlying theory where $\sum_g d_g g \partial_g + \sum_\sigma d_\sigma \sigma \partial_\sigma$ is the sum of all the canonical scaling transformations with respect to the parameters $[g; \sigma]$.

New forms of CSE with underlying structures

Now, for a general n-point Green function $\Gamma^{(n)}$ (that is at least connected in the sense of Feynman diagrams), the differential form for canonical scaling laws with underlying structures should read

$$\left\{\lambda\partial_\lambda + \sum_g d_g g \partial_g + \sum_\sigma d_\sigma \sigma \partial_\sigma - d_{\Gamma^{(n)}}\right\} \Gamma^{(n)}([\lambda p], [g; \sigma]) = 0. \tag{3}$$

Since $\sum d_g g \partial_g$ inserts $-\mathrm{i}\int \mathrm{d}^D x \Theta(x)$, the alternative form of equation (3) reads

$$\left\{\lambda\partial_\lambda + \sum_\sigma d_\sigma \sigma \partial_\sigma - d_{\Gamma^{(n)}}\right\} \Gamma^{(n)}([\lambda p], [g; \sigma]) = \mathrm{i}\Gamma^{(n)}_\Theta([0; \lambda p], [g; \sigma]). \tag{4}$$

This latter equation just parallels the conventional CSE. To obtain the conventional form of CSE, the next step is to expand $\sum_\sigma d_\sigma \sigma \partial_\sigma$ into the sum of insertion of various EFT operators with the associated 'anomalous' dimensions. It is easy to see that, this expansion itself is exactly the general version of the renormalization group [8], which is a 'decoupling' theorem in the underlying theory's terminology.

Before elaborating on the EFT limit, we rewrite the CSE with underlying structures (4) in the following concise form,

$$\{\lambda\partial_\lambda - d_{\Gamma^{(n)}}\}\Gamma^{(n)}([\lambda p], [g; \sigma]) = \mathrm{i}\Gamma^{(n)}_{\tilde{\Theta}}([0; \lambda p], [g; \sigma]), \tag{5}$$

with $\tilde{\Theta}$ being given in equation (2). This is the basis for our rederivation of the low-energy theorems.

In the EFT limit, the underlying parameters' contributions should be replaced by appropriate 'agent' constants $[\bar{c}]$, at least to balance the dimensions in necessary places. Then canonical scaling $\sum_\sigma d_\sigma \sigma \partial_\sigma$ is replaced by $\sum_{\bar{c}} d_{\bar{c}} \bar{c} \partial_{\bar{c}}$, which in turn induces the insertion of the EFT operators $([I_{O_i}])$ accompanied with appropriate 'anomalous' dimensions. In formula, this is the 'decoupling' theorem of the underlying structures (we use $\check{P}_{\mathrm{EFT}}$ to symbolize the delicate EFT limit operation)

$$\check{P}_{\mathrm{EFT}}\left\{\sum_\sigma d_\sigma \sigma \partial_\sigma [\cdots]\right\} = \sum_{\bar{c}} d_{\bar{c}} \bar{c} \partial_{\bar{c}} [\cdots] = \sum_{O_i} \delta_{O_i} I_{O_i} [\cdots], \tag{6}$$

with I_{O_i} denoting the insertion of the EFT operator O_i and δ_{O_i} the corresponding 'anomalous dimension' that must be a function of the EFT couplings $[g]$ and the 'agent' constants $\{\bar{c}\}$. For further discussion of the EFT contents of this expansion, please refer to [8]. Consequently, the complete trace operator $\tilde{\Theta}$ now becomes, $\tilde{\Theta} = \Theta + \Delta\Theta \Rightarrow \mathrm{i}\{\sum_g d_g g \partial_g + \sum_{O_i} \delta_{O_i} I_{O_i}\}$ with $\Delta\Theta$ being now trace anomalies in terms of EFT operators, $\sum_{O_i} \delta_{O_i} O_i$.

Introducing the operator $\hat{I}_{i\tilde{\Theta}} \equiv -\sum_g d_g g \partial_g - \sum_{O_i} \delta_{O_i} I_{O_i}$, equations (3) and (5) take the following concise forms:

$$\{\lambda\partial_\lambda - \hat{I}_{\mathrm{i}\tilde{\Theta}} - d_{\Gamma^{(n)}}\}\Gamma^{(n)}([\lambda p], [g; \bar{c}]) = 0, \tag{7}$$

$$\{\lambda\partial_\lambda - d_{\Gamma^{(n)}}\}\Gamma^{(n)}([\lambda p], [g; \bar{c}]) = \mathrm{i}\Gamma^{(n)}_{\tilde{\Theta}}([\lambda p], [g; \bar{c}]). \tag{8}$$

For the cases with composite operators, we have

$$\{\lambda\partial_\lambda - \hat{I}_{i\tilde{\Theta}} - d_\Gamma\}\Gamma^{(n)}_{O_A,\ldots}([\lambda p], [\lambda p_A, \ldots], [g; \bar{c}]) = 0; \tag{9}$$

$$\{\lambda\partial_\lambda - d_\Gamma\}\Gamma^{(n)}_{O_A,\ldots}([\lambda p], [\lambda p_A, \ldots], [g; \bar{c}]) = \mathrm{i}\Gamma^{(n)}_{\tilde{\Theta}, O_A,\ldots}([0; \lambda p], [\lambda p_A, \ldots], [g; \bar{c}]). \tag{10}$$

At this stage, we should recall that, for a generic EFT, there might be some trace anomalies from the renormalization of composite operators in $\tilde{\Theta}$, i.e., in the contents of $\sum_{O_i} \delta_{O_i} O_i$ [8].

Here some remarks are in order. In the new versions of CSE, all the renormalization/factorization procedures are understood to be accomplished in the underlying theory's point of view. Alternatively, one could view the presence of the underlying parameters $[\sigma]$ or their 'agents' $[\bar{c}]$ as a certain prescription of a consistent regularization/renormalization to be specified, provided the EFT could be consistently renormalized. Thus, a renormalization prescription only affects EFT through the trace anomalies and the presence of the 'agents'. All the objects to be discussed below are understood to be already renormalized or rendered well defined in the sense of underlying theory. For CSE, all the nontrivial effects from renormalization are accommodated in the trace anomalies, i.e., effected through the operation $\hat{I}_{i\tilde{\Theta}}$. We should also recall that the foregoing derivation is not aiming at new results, but simply to demonstrate that the conventional RGE and CSE could allow for a very simple and natural interpretation from the viewpoint of the complete theory that is well defined with the structures underlying the effective theories.

For the purpose below, one could also resort to the conventional CSE (in any specific scheme) and turn it into the form of equation (10), bypassing the underlying theory viewpoint advocated above. For example, in QED, this is to replace the conventional CSE

$$\{\lambda\partial_\lambda - \beta_\alpha\partial_\alpha + m(1 + \gamma_m)\partial_m + n_A\gamma_A + n_\psi\gamma_\psi - d_{\Gamma^{(n)}}\}\Gamma^{(n)}([\lambda p], [\alpha, m; \mu]) = 0,$$

with the operator insertion version

$$\{\lambda\partial_\lambda - \hat{I}_{i\tilde{\Theta}} - d_{\Gamma^{(n)}}\}\Gamma^{(n)}([\lambda p], [\alpha, m; \mu]) = 0,$$

or equivalently,

$$\{\lambda\partial_\lambda - d_{\Gamma^{(n)}}\}\Gamma^{(n)}([\lambda p], [\alpha, m; \mu]) = \mathrm{i}\Gamma^{(n)}_{\tilde{\Theta}}([0; \lambda p], [\alpha, m; \mu]). \tag{11}$$

Here, $\tilde{\Theta} = \frac{\beta_\alpha}{4\alpha}F^2 + (1 + \gamma_m)m\bar{\psi}\psi - 2\gamma_\psi\bar{\psi}\mathrm{i}\partial\!\!\!/\psi = \frac{\beta_\alpha}{4\alpha}F^2 + (1 + \bar{\gamma}_m)m\bar{\psi}\psi$ due to equations of motion. (Note that, in QED, $\frac{\beta_\alpha}{\alpha} = 2\gamma_A$.) Such a concise form is more useful for our purpose. What we did in the forgoing paragraphs is just the reformulation of such conventional CSE's in any consistent EFT in terms of the underlying theory's perspective. The conclusions obtained below remain the same if one adopts the conventional CSE's after turning them into the form like equation (10) or (11).

Low-energy theorems with trace anomalies

Now, we are ready to rederive or prove the low-energy theorems first advanced and proved in [1, 2], using the new versions of CSE given above. Again, all the objects involved below are understood as being at least connected in the sense of Feynman diagrams.

First we consider a general EFT operator O. Since $\Gamma_O \equiv \langle \text{vac}|O(0)|\text{vac}\rangle$ is independent of momentum, then equation (10) for this object reduces to the following simple one,

$$-d_{\Gamma_O}\Gamma_O([g;\bar{c}]) = \mathrm{i}\Gamma_{\tilde{\Theta},O}([g;\bar{c}]), \tag{12}$$

as $\lambda\partial_\lambda\Gamma_O = 0$. That is, $(d_O = d_{\Gamma_O})$,

$$-d_O\langle \text{vac}|O(0)|\text{vac}\rangle = \hat{I}_{i\tilde{\Theta}}\langle \text{vac}|O(0)|\text{vac}\rangle = \mathrm{i}\int \mathrm{d}^D x\langle \text{vac}|T\{\tilde{\Theta}(x)O(0)\}|\text{vac}\rangle. \tag{13}$$

Assuming that the vacuum state is translationally invariant, we arrive at the following familiar form [2]:

$$\mathrm{i}\int \mathrm{d}^D x\langle \text{vac}|T\{O(x)\tilde{\Theta}(0)\}|\text{vac}\rangle = -d_O\langle \text{vac}|O(0)|\text{vac}\rangle. \tag{14}$$

Noting that the left-hand side of equation (14) is the low-energy limit of the correlation function $\Pi_{\tilde{\Theta}O}(Q) \equiv \mathrm{i}\int \mathrm{d}^D x\, \mathrm{e}^{\mathrm{i}qx}\langle \text{vac}|T\{O(x)\tilde{\Theta}(0)\}|\text{vac}\rangle$, the low-energy theorem

$$\Pi_{\tilde{\Theta}O}(0) = -d_O\langle \text{vac}|O(0)|\text{vac}\rangle \tag{15}$$

follows immediately. It is just an implicit form of the CSE for the vertex function $\Gamma_O \equiv \langle \text{vac}|O(0)|\text{vac}\rangle$. Here, we recall that the trace operator $\tilde{\Theta}$ contains all the sources that break the scale invariance, both canonical (masses) and anomalous ones. For instance, the trace operator $\tilde{\Theta}$ of QCD with massive quarks contains the canonical quark mass operator $\sum_f m_f\bar{q}_f q_f$ besides the scale or trace anomalies from gluon fields $\left(\frac{\beta(g_s)}{4g_s}G^a_{\mu\nu}G^{a\mu\nu}\right)$ and quark fields $\left(\sum_f m_f\bar{\gamma}_{m_f}\bar{q}_f q_f\right)$. In [1, 2], the low-energy theorem was derived in QCD for the trace anomaly from $\frac{\beta(g_s)}{4g_s}G^a_{\mu\nu}G^{a\mu\nu}$; the contributions from $\sum_f m_f(1+\bar{\gamma}_{m_f})\bar{q}_f q_f$ were moved to another side as terms that are formally linear in quark masses. That is, denoting $\tilde{\Theta}_{\text{gluon}} \equiv \frac{\beta(g_s)}{4g_s}G^a_{\mu\nu}G^{a\mu\nu}$ and moving $\sum_f m_f(1+\bar{\gamma}_{m_f})\bar{q}_f q_f$ to the right-hand side, equation (15) for QCD could be cast into the following form,

$$\Pi_{\tilde{\Theta}_{\text{gluon}}O}(0) = -d_O\langle \text{vac}|O(0)|\text{vac}\rangle[1+\mathcal{O}(m)], \tag{16}$$

which is exactly equation (52) in [2].

In pure four-dimensional gluodynamics, for $O = \tilde{\Theta} = \frac{\beta(g_s)}{4g_s}G^a_{\mu\nu}G^{a\mu\nu}$, the above low-energy theorem reads $(d_{\tilde{\Theta}} = 4)$

$$\Pi_{\tilde{\Theta}\tilde{\Theta}}(0) = -4\langle \text{vac}|\tilde{\Theta}|\text{vac}\rangle. \tag{17}$$

This is the important relation that has been extensively exploited in various QCD (including SUSYQCD) and hadron studies [3–5].

Applying the operation $\hat{I}_{i\tilde{\Theta}}n$ times, we could obtain identities [2, 7]

$$\begin{aligned}(\hat{I}_{i\tilde{\Theta}})^n\langle \text{vac}|O(0)|\text{vac}\rangle &= i^n\int\prod_{i=1}^n \mathrm{d}^D x_i\langle \text{vac}|T\left\{\prod_{i=1}^n(\tilde{\Theta}(x_i))O(0)\right\}|\text{vac}\rangle\\ &= (-d_O)^n\langle \text{vac}|O(0)|\text{vac}\rangle.\end{aligned} \tag{18}$$

These relations could be seen as corollaries of CSE.

Next, we would like to show that the low-energy theorems for the amplitudes with non-vanishing momentum given in [7] are just the alternative forms of the new version of CSE, equation (10). These low-energy theorems are the following relations,

$$\Pi_3 = 2\frac{\mathrm{d}\Pi_2}{\mathrm{d}\ln Q^2} - (2d_O - D)\Pi_2, \tag{19}$$

$$\Pi_4 = 4\frac{\mathrm{d}^2\Pi_2}{(\mathrm{d}\ln Q^2)^2} - 4(2d_O - D)\frac{\mathrm{d}\Pi_2}{\mathrm{d}\ln Q^2} + (2d_O - D)^2\Pi_2, \tag{20}$$

and so on, where

$$\Pi_k = i^{k-1}\int \mathrm{d}^D x_1 \cdots \mathrm{d}^D x_{k-1}\, \mathrm{e}^{\mathrm{i}qx_1}\langle \mathrm{vac}|T\{O(x_1)O(0)\tilde{\Theta}(x_2)\cdots\tilde{\Theta}(x_{k-1})\}|\mathrm{vac}\rangle, \qquad k \geqslant 2, \tag{21}$$

and $Q^2 = -q^2$.

Again, we note that in the following and also in the foregoing derivations, the contents of the trace operator $\tilde{\Theta}(x_i)$ will not be specified, hence it is applicable to both QCD and other EFT's.

To proceed, we first note that, using our notations, the left-hand sides of equations (19), (20) are just $\hat{I}_{i\tilde{\Theta}}\Pi_2, \hat{I}_{i\tilde{\Theta}}\Pi_3$, respectively. Next, we note that $2\frac{\mathrm{d}\Pi_2}{\mathrm{d}\ln Q^2} = \lambda\partial_\lambda\Pi_2(\lambda Q)$. Since $2d_O - D = d_{\Pi_2}$, so equation (19) becomes

$$\hat{I}_{i\tilde{\Theta}}\Pi_2 = \mathrm{i}\Pi_{\tilde{\Theta},2} = (\lambda\partial_\lambda - d_{\Pi_2})\Pi_2. \tag{22}$$

This is nothing but the CSE (equation (10)) for Π_2.

To turn equation (20) into the form of CSE, we employ equation (19) to replace each $2\frac{\mathrm{d}\Pi_2}{\mathrm{d}\ln Q^2}$ term in equation (20) with $\Pi_3 + d_{\Pi_2}\Pi_2$ repeatedly till all such terms in equation (20) are replaced. Then we end up with

$$\hat{I}_{i\tilde{\Theta}}\Pi_3 = 2\frac{\mathrm{d}\Pi_3}{\mathrm{d}\ln Q^2} - d_{\Pi_3}\Pi_3 = (\lambda\partial_\lambda - d_{\Pi_3})\Pi_3, \tag{23}$$

where obviously, $d_{\Pi_3} = d_{\Pi_2} = 2d_O - D$. This is again a CSE.

In general, we have the following CSE:

$$\Pi_{k+1} = \hat{I}_{i\tilde{\Theta}}\Pi_k = \left(\lambda\partial_\lambda - d_{\Pi_k}\right)\Pi_k, \qquad d_{\Pi_k} = 2d_O - D, \qquad \forall k \geqslant 2. \tag{24}$$

It is of course possible to derive more low-energy theorems involving trace anomalies from the new form of CSE, equation (10), by studying the low-energy limits of various objects so that $\lambda\partial_\lambda$ does not contribute.

Discussions and summary

In the above deductions, we have deliberately been not specific about the concrete contents of the EFT trace operator and trace anomalies. Therefore, our formulation and derivation are valid for any kind of EFT, as long as it could be consistently regularized and renormalized/factorized. One could well apply such low energy theorems in the EFT's other than QCD. It would be especially interesting to consider its possible implications for the low-energy EFT's of QCD and/or the electroweak sectors of the standard model.

In [4], the low-energy theorem involving trace anomaly in QCD has been exploited to predict 'soft' pomeron in an impressive manner. This nonperturbative approach to hadronic interactions at high energy and small momentum transfer is directly based on trace anomaly of QCD. With the direct connections between CSE and the low-energy theorems of QCD being revealed, and the interesting relations between the low-energy theorem approach and JIMWLK approach [9] (as an evolution equation) being pointed out in [4], we feel it an interesting attempt to explore the possible connections or interplay between CSE and JIMWLK. We would like to note that the new form of CSE we used here looks similar to the JIMWLK equation in the sense that the scale 'evolution' $\lambda\partial_\lambda$ of CSE is governed by the operator $\hat{I}_{i\tilde{\Theta}}$, while the rapidity evolution of JIMWLK is governed by the corresponding operator defined in [9].

Of course, the CSE's describe the full scaling laws for an EFT with respect to all the active dynamical parameters, not a specific or partial evolution along certain dynamical variable (say, longitudinal momentum fraction x or transverse momentum squared Q^2). Nevertheless, it is natural to introduce the 'running' of the couplings of the EFT operators appearing in the trace anomalies, i.e., $\delta_{O_i} O_i$, as is already explicated in [8]. These 'running' objects obey the corresponding evolution equations introduced using Coleman's bacteria analogue [10] with the 'anomalous dimensions' (or, in a sense, the evolution kernels) given by the 'decoupling theorem' or EFT expansion described by equation (6). It would be interesting to explore if there is any hidden relation between the full scaling laws encoded in CSE and the well-known evolution equations in QCD like DGLAP [11]. At least there is a conceptual link: the treatment of soft and collinear singularities would inevitably lead to the introduction of new scales (e.g., factorization scales) or dimensional parameters that should somehow contribute to the trace anomalies. Since these evolution equations prompt the definition of certain nonperturbative objects (PDF, or matrix elements between hadronic states), examining these equations from the perspective of CSE as the full scaling laws should be helpful in clarifying the overall structure of QCD and the like, especially in delineating the delicate interplay between the perturbative and nonperturbative sectors. Investigations of such possibilities will be pursued in the future. It is also in conformity with the recent efforts using renormalization group methods to resum various large logarithms in order to avoid certain pathologies like Landau-pole singularity in the conventional resummation approaches [12].

In summary, we presented some new forms of Callan–Symanzik equations and showed that the important low-energy theorems involving trace anomalies à la NSVZ follow as immediate consequences of the new forms of CSE. In other words, these theorems were proved in a simple and general manner so that they are valid in any consistent EFT's. The possible relations between CSE and various QCD evolution equations and other possible applications of the CSE were briefly discussed.

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